Spaces of continuous and holomorphic functions with weight conditions

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We survey some properties of spaces of continuous and holomorphic functions the topology of which is given by weighted sup-seminorms. The building blocks are the spaces
$Cv(X) := \{f$ continuous complex valued on $X;\ p_v(f) = \sup_X v|f| < \infty\}$,
$Cv_0(X) := \{f \in Cv(X);\ vf$ vanishes at infinity on $X\}$,
where X is a completely regular Hausdorff space and v is a nonnegative upper semicontinuous "weight", resp. the corresponding spaces $Hv(G)$, $Hv_0(G)$ of holomorphic functions on an open set G in N complex dimensions.

Intersections of the building blocks lead to Nachbin's weighted spaces $CV(X)$ and $CV_0(X)$. Many examples can be given. Nachbin used the spaces to discuss the weighted approximation problem and to prove (for $CV_0(X)$) generalizations of the Stone-Weierstrass theorem and of Bishop's theorem in approximation theory.

Countable unions of the building blocks, endowed with the locally convex inductive limit topology, are the so-called weighted inductive limits $\mathcal{V}C(X)$ and $\mathcal{V}_0C(G)$. Their holomorphic counterparts $\mathcal{V}H(G)$ and $\mathcal{V}_0H(G)$ occur in many applications. Together with R. Meise and W.H. Summers, projective hulls of these spaces were introduced, and the projective description problem was studied in order to get an explicit formula for a basis of the continuous seminorms of the inductive limit topology.

The following two references only mark the beginning of long developments:

Leopoldo Nachbin, Elements of approximation theory, xii + 119 pp., Reprint of the 1967 edition, Krieger 1976

Klaus D. Bierstedt, Reinhold Meise, William H. Summers, A projective description of weighted inductive limits, Trans. Amer. Math. Soc. **272**, 107–160 (1982)