SEMISIMPLE ORBITAL INTEGRALS IN THE SYMPLECTIC SPACE, FOR A REAL REDUCTIVE PAIR

MARK MCKEE AND TOMASZ PRZEBINDA (UNIVERSITY OF OKLAHOMA)

Let W be a finite dimensional vector space over $\mathbb{R}$ with a non-degenerate symplectic form. Denote by Sp the corresponding symplectic group and by $\widetilde{\mathrm{Sp}}$ the metaplectic group.

Let $\mathrm{G}, \mathrm{G}' \subseteq \mathrm{Sp}$ be a dual pair. Denote by $\widetilde{G}, \widetilde{G}'$ the preimages of G and G' in $\widetilde{\mathrm{Sp}}$. Each representation $\Pi \otimes \Pi'$ of $\widetilde{G} \times \widetilde{G}'$, in Howe's correspondence, [How89], is determined by a unique up to a scalar multiple distribution $f = f_{\Pi\otimes\Pi'} \in S^*(\mathsf{W})$, called the intertwining distribution in [Prz93].

The intertwining distribution f is $\mathrm{G} \times \mathrm{G}'$-invariant (under the permutation representation of this group on the symplectic space W) and is an eigendistribution for the action of the center of the universal enveloping algebra of G (or equivalently G') on $S^*(\mathsf{W})$, via twisted convolution. Thus, f is an invariant eigen-distribution on the symplectic space W.

As explained in [Prz06], one may view each dual pair G, G' as a supergroup. Then W is the odd part in the corresponding Lie superalgebra. In this context we have the semisimple elements of W and Cartan subspaces $\mathfrak{h}_{\bar{1}} \subseteq \mathsf{W}$. (We exclude the case $G = \mathrm{Sp}$, $G' = \{\pm 1\}$ where the only semisimple element of W is zero.) Each semisimple orbit passes through a Cartan subspace and the intersection of the two is an orbit of the corresponding (finite) Weyl group. The set of the regular semisimple elements is dense in W and the restriction of f to that set is a smooth function. In this setting we would like to follow Harish-Chandra's theory of the invariant eigen-distributions on a reductive Lie algebra, [Har65]. The project is far from completion, but at this initial stage, we notice some similarities and some differences.

As an invariant distribution, f factors through the orbital integrals through each Cartan subspace. We show that any such orbital integral of a rapidly decreasing function on W is rapidly decreasing at infinity on $\mathfrak{h}_{\bar{1}}$, though it might have logarithmic growth at some singular points. In particular it is not necessarily true that a semisimple orbital integral of a Schwartz function on the symplectic space is a Schwartz function on the corresponding Cartan subspace, though it is certainly differentiable on the regular set. Nevertheless, these estimates suffice to produce intertwining distributions in many interesting cases, which we shall explain.

References

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Department of Mathematics, University of Oklahoma, Norman, OK 73019
E-mail address: mmckee@math.ou.edu, tprzebinda@gmail.com